(b) Use dynamic programming to solve the following problem :

Maximize $Z = 2X_1 + 4X_2$

subject to the constraints

$$2X_1 + 3X_2 \leq 48,$$

$$X_1 + 3X_2 \leq 42,$$

$$X_1 + X_2 \leq 21,$$

and $X_1, X_2 \geq 0.$ 10+10=20

Exam. Code : 211003
Subject Code : 4971

M.Sc. Mathematics 3rd Semester (Batch 2020-22)

OPERATIONS RESEARCH—I

Paper—MATH-578

Time Allowed—3 Hours] [Maximum Marks—100

Note :—Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

I. (a) Solve the linear programming problem using two phase simplex method :

Maximize $Z = -4X_1 - 3X_2 - 9X_3$

subject to the constraints

$$2X_1 + 4X_2 + 6X_3 \geq 15,$$

$$6X_1 + X_2 + 6X_3 \geq 12,$$

and $X_1, X_2, X_3 \geq 0.$

(b) Use M-method to solve the following Linear Programming Problem :

Maximize $Z = 6X_1 + 4X_2$

subject to the constraints

$2X_1 + 3X_2 \leq 30,$

$3X_1 + 2X_2 \leq 24,$

and $X_1 + X_2 \geq 3$ and $X_1, X_2 \geq 0.$

10+10=20

II. (a) Solve the Linear Programming Problem :

Maximize $Z = 3X_1 + 2X_2$

subject to the constraints

$2X_1 + X_2 \leq 2,$

$3X_1 + 4X_2 \geq 12,$

and $X_1, X_2 \geq 0.$

Also find the alternative optimum solution if exists.

(b) Solve the following linear programming problem by the simplex method :

Minimize $Z = 4X_1 + 6X_2$

subject to the constraints

$-2X_1 + 3X_2 = 3,$

$4X_1 + 5X_2 \geq 10,$

$4X_1 + 8X_2 \geq 5,$

and $X_1, X_2 \geq 0.$ 10+10=20

SECTION—B

III. (a) Prove that the dual of a given primal, is the primal itself.

(b) Determine the optimal transportation plan from the following table, given the plant to market shipping costs and quantities at each market and available at each plant :

Plant	W-1	W-2	W-3	W-4	Availability
F-1	11	20	7	8	50
F-2	21	16	10	12	40
F-3	8	12	18	9	70
Requirement	30	25	35	40	

10+10=20

IV. (a) Consider the following Linear Programming Problem :

Maximize $Z = X_1 + 5X_2 + 3X_3$

subject to the constraints

$X_1 + 2X_2 + X_3 = 3,$

$2X_1 - X_2 = 4,$

and $X_1, X_2, X_3 \geq 0.$

Write the associated dual problem and determine its optimal solution.

(b) The following table provides all the necessary information on the availability of supply to each warehouse, the requirement of each market, and the unit transportation cost (in Rupees) from each warehouse to each market. The shipping clerk of the shipping agency has worked out the following schedule, based on his own experience : 12 units from **A** to **Q**, 1 unit from **A** to **R**, 9 units from **A** to **S**, 15 units from **B** to **R**, 7 units from **C** to **P** and 1 unit from **C** to **R**.

(i) Find the optimal schedule and minimum total transport cost.

(ii) If the clerk is approached by a carrier of route **C** to **Q**, who offers to reduce his rate in the hope of getting some business, by how much should the rate be reduced before the clerk would offer him the business.

		Market				
		P	**Q**	**R**	**S**	**Supply**
Warehouse	**A**	6	3	5	4	**22**
	B	5	9	2	7	**15**
	C	5	7	8	6	**8**
	Demand	**7**	**12**	**17**	**9**	**45**

10+10=20

SECTION—C

V. (a) The Secretary of a school is taking bids on the city's four school bus routes. Four companies have made the bids (in Rupees), as detailed in the following table :

	Route-1	**Route-2**	**Route-3**	**Route-4**
Bus-1	4000	5000	–	–
Bus-2	–	4000	–	4000
Bus-3	3000	–	2000	–
Bus-4	–	–	4000	5000

Suppose each bidder can be assigned only one route. Use the assignment model to minimize the school's cost of running the four bus routes.

(b) In a game of matching coins with two players, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and losses 1/2 unit of value when there is one head and one tail. Determine the payoff matrix, the best strategies for each player and the value of the game to A. 10+10=20

VI. (a) There are four vendors located in different parts of Delhi (A, B, C and D) who have to supplied ice-cream every day. The following matrix displays the distances (in kms) between the depot and the four vendors. What route should the company van follow so that the total distance travelled is minimized ?

Depot	Vendor-A	Vendor-B	Vendor-C	Vendor-D
Depot	–	3.5	3	4	2
Vendor-A	3.5	–	4	2.5	3
Vendor-B	3	4	–	4.5	3.5
Vendor-C	4	2.5	4.5	–	4
Vendor-D	2	3	3.5	4	–

(b) Obtain the optimal strategies for both persons and the value of the game for two-person zero-sum game whose payoff matrix is as follows :

	Player–B-1	Player–B-2
Player–A-1	1	–3
Player–A-2	3	5
Player–A-3	–1	6
Player–A-4	4	1
Player–A-5	2	2
Player–A-6	–5	0

10+10=20

SECTION—D

VII. (a) Solve the integer programming problem using Gomory's cutting plane method :

Maximize $Z = X_1 + 4X_2$

subject to the constraints

$2X_1 + 4X_2 \leq 5$,

$5X_1 + 3X_2 \leq 15$,

and $X_1, X_2 \geq 0$ are integers.

(b) Use dynamic programming to solve the following problem :

Maximize $Z = 8X_1 + 7X_2$

subject to the constraints

$2X_1 + X_2 \leq 8$,

$5X_1 + 2X_2 \leq 15$,

and $X_1, X_2 \geq 0$. 10+10=20

VIII. (a) Solve the all integer programming problem using the branch and bound method :

Minimize $Z = 3X_1 + 2.5X_2$

subject to the constraints

$X_1 + 2X_2 \geq 20$,

$3X_1 + 2X_2 \geq 50$,

and $X_1, X_2 \geq 0$ are integers.